A General Lattice Model for Merging Symbolic Execution Branches

Dominic Scheurer

scheurer@cs.tu-darmstadt.de

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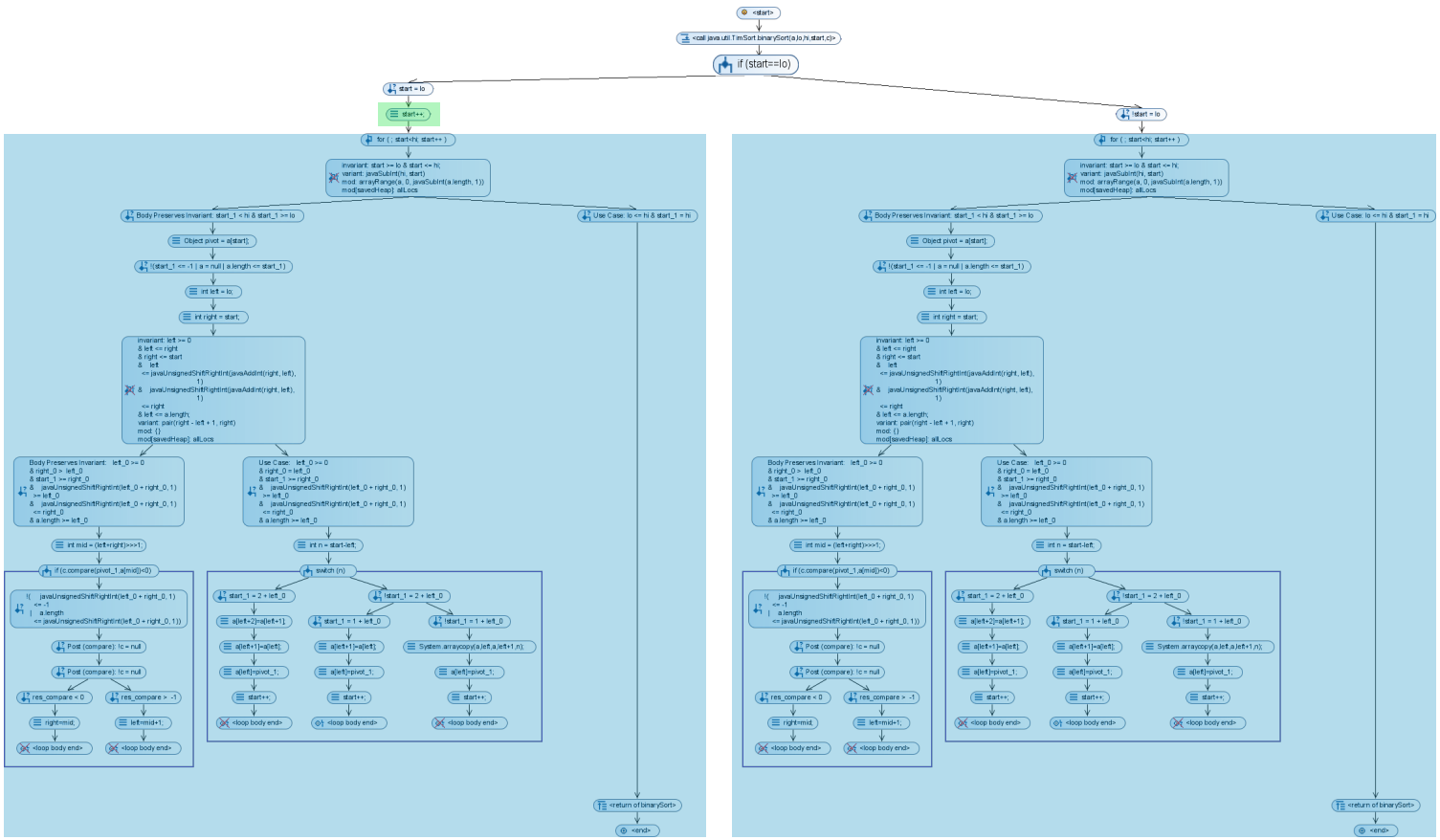
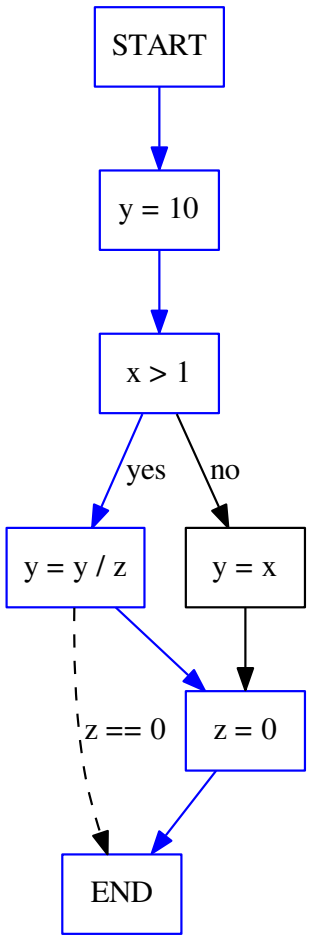
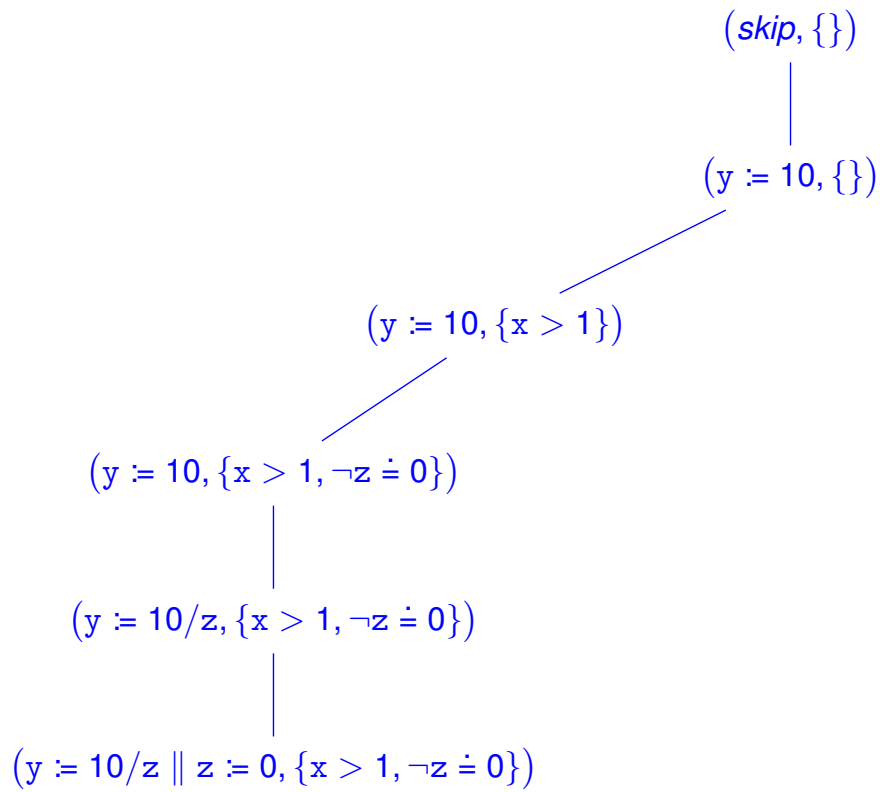


Symbolic Execution follows all possible execution paths





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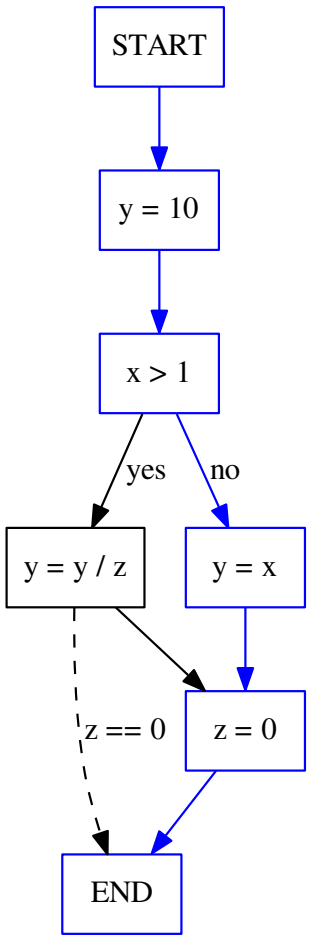

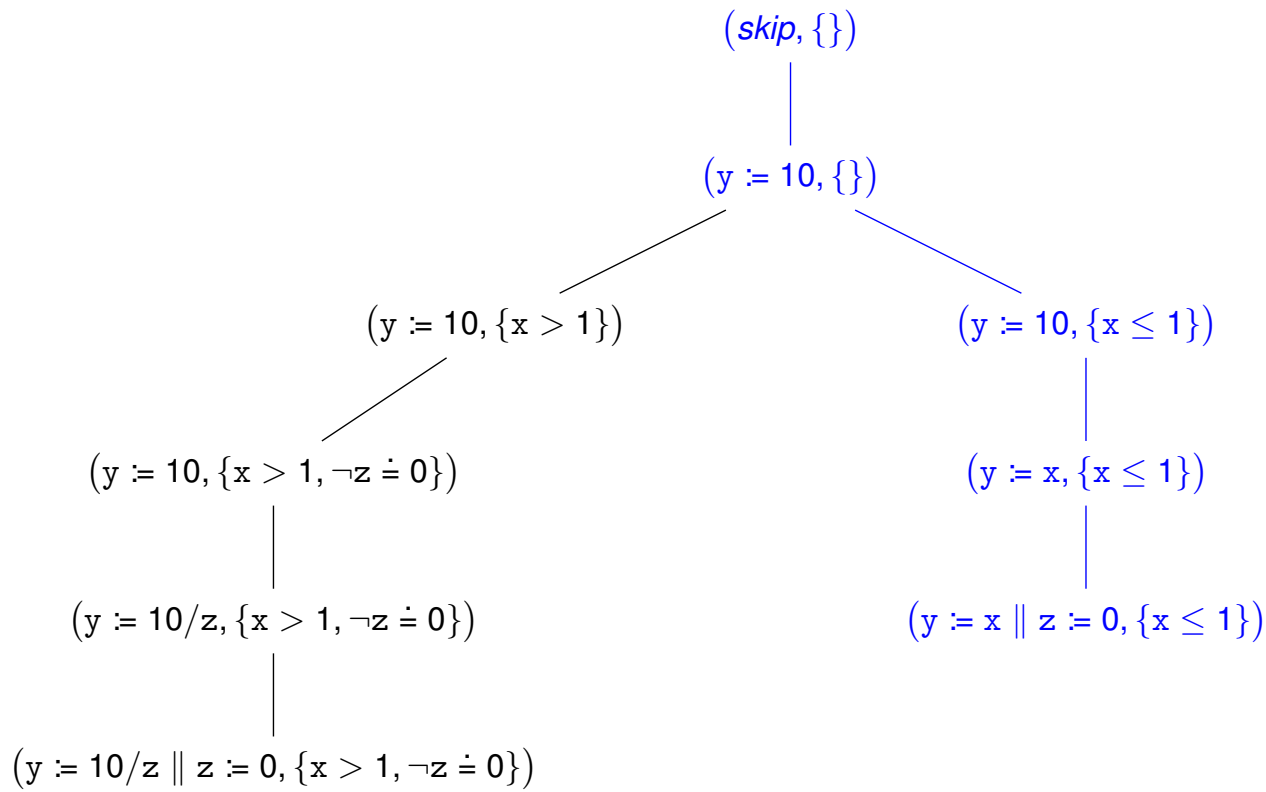

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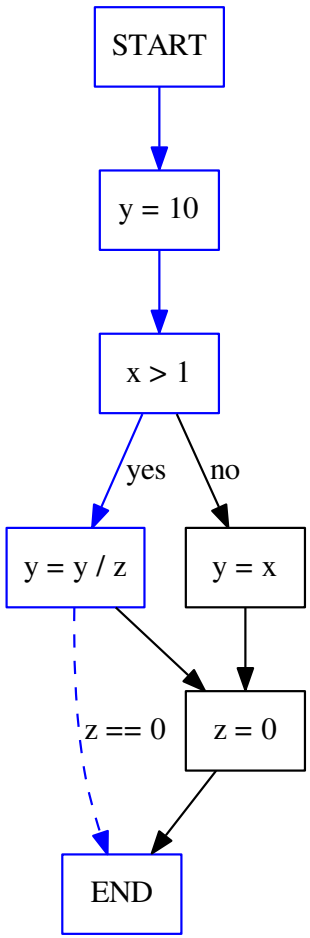

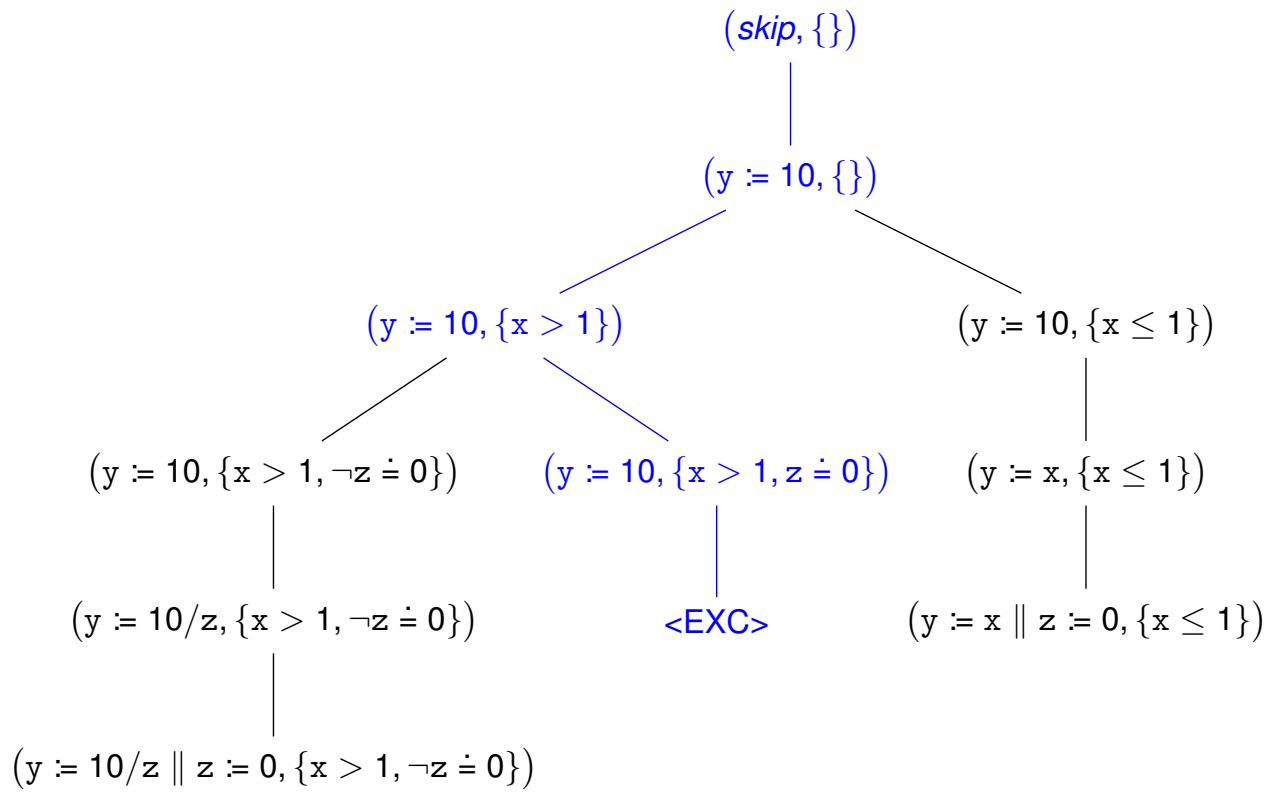

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Beware of the *Path Explosion Problem*! Example: TimSort.binarySort

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```
private static void binarySort(Object[] a, /* ... */) {
    assert lo <= start && start <= hi;

    if (start == lo)
        start++;

    for ( ; start < hi; start++) {
        // ...
        while (left < right) { /* ... */ }
        // ...

        int n = start - left;
        switch (n) { /* ... */ }
        a[left] = pivot;
    }
}
```

Beware of the *Path Explosion Problem*!
TimSort.binarySort – Symbolic Execution Tree

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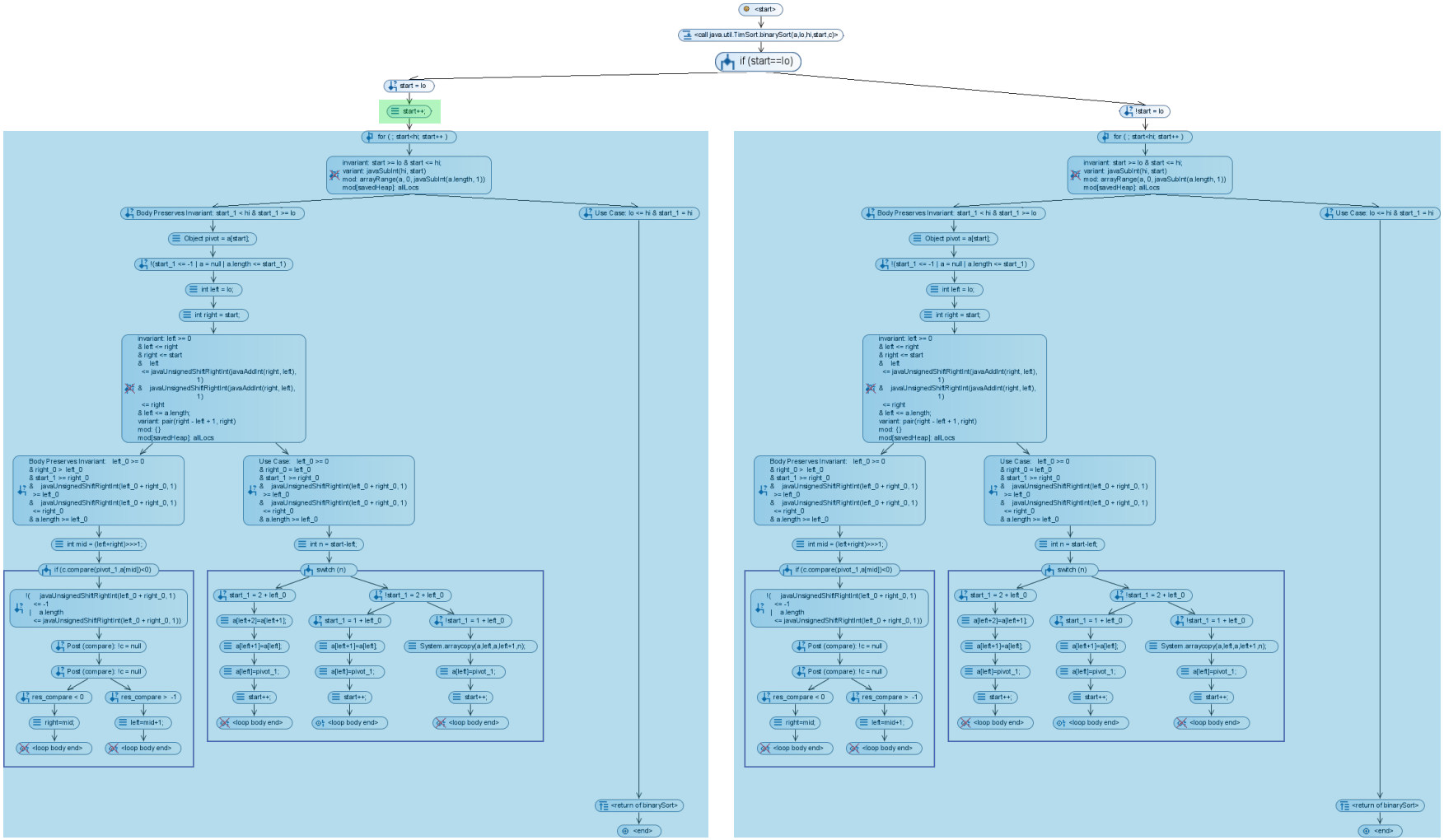

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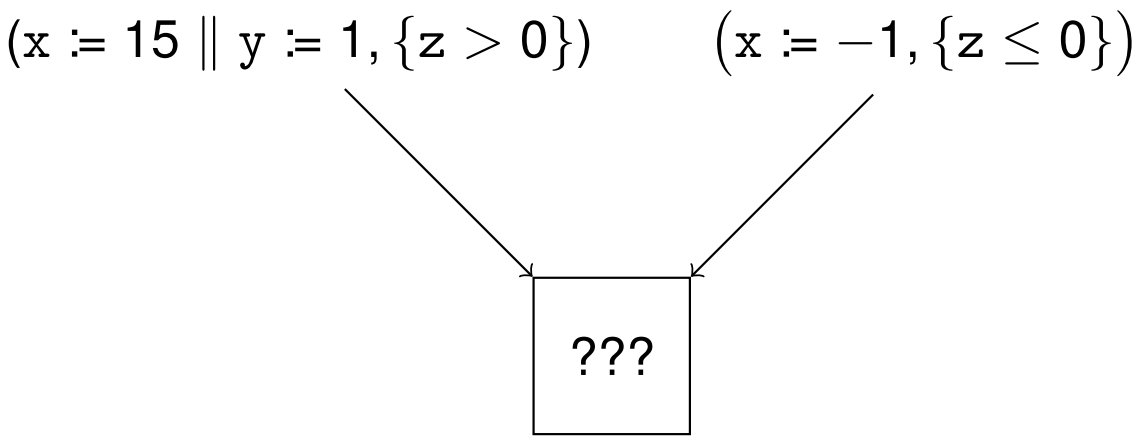
Goals



TimSort.binarySort – Proof statistics after SE

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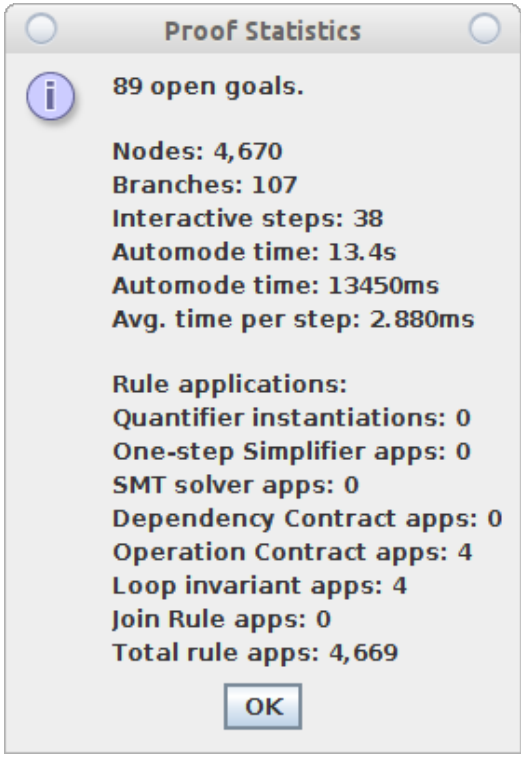

Statistics after finishing Symbolic Execution

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TimSort.binarySort – Proof statistics after SE

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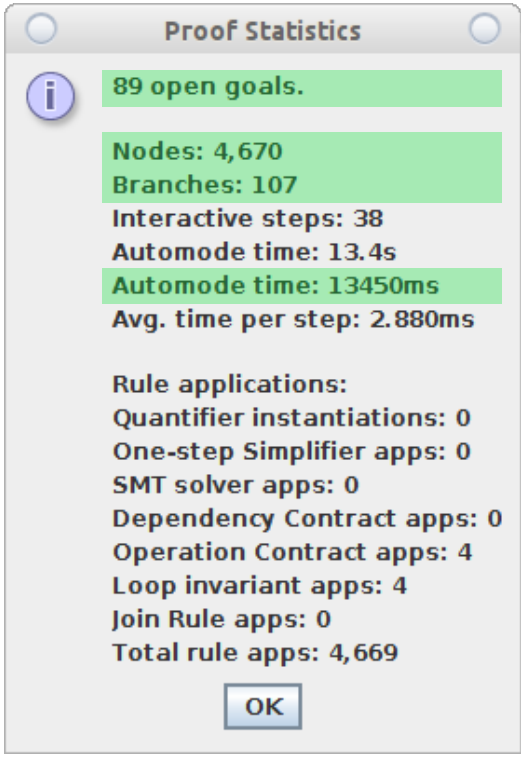

Statistics after finishing Symbolic Execution

Statistics after finishing Symbolic Execution *with one merge*

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TimSort.binarySort – Proof statistics after SE

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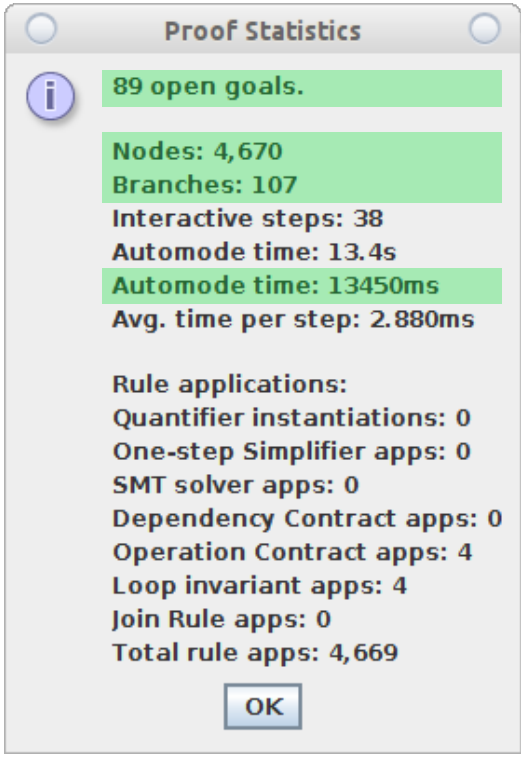

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Preliminary Considerations

Java Dynamic Logic (Java DL): A program logic for proving functional properties about Java programs

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Syntax

$$\mathtt{x} = \mathit{true} \rightarrow \underbrace{\left\{ \mathtt{y} := c \parallel \underbrace{\mathtt{x} := \mathit{if}\,(\varphi)\,\mathit{then}\,(1)\,\mathit{else}\,(-c)}_{\text{elementary update}} \right\}}_{\text{parallel update}} \underbrace{\langle \mathtt{y}{+}{+}; \mathtt{return};\rangle\, \mathtt{y} > 0}_{\text{Modality}}$$

Semantics

$$\mathit{val}\left(K, \sigma; \{\mathtt{y} := 1\}\right) = \sigma_{[\mathtt{y} \mapsto 1]}$$

$$\mathit{val}\,(K, \sigma; \exists x; x > 0) = \mathit{tt} \iff (K, \sigma) \models \exists x; x > 0$$

Sequent Calculus

$$\frac{\Delta \implies \Gamma, \{\mathtt{y} := 1\}\,\langle p \rangle\,\varphi}{\Delta \implies \Gamma, \langle \mathtt{y} = 1;\ p \rangle\,\varphi}\ \text{assignment}$$

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An SE state is a triple

corresponding to a Java DL sequent

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SE State $(\underbrace{\mathtt{x} := 15 \parallel \mathtt{y} := 1}_{U}, \underbrace{\{\mathtt{z} > 0, !(\mathtt{o} \doteq \mathit{null})\}}_{C}, \underbrace{\langle p \rangle \varphi}_{\text{PC}})$

Symbolic State U: current value of program variables

Path Condition C: (Set of) constraints leading to the current execution path

Program Counter: remaining program to execute p and post condition φ

Sequent $\bigwedge C \Longrightarrow \{U\} \langle p \rangle \varphi$

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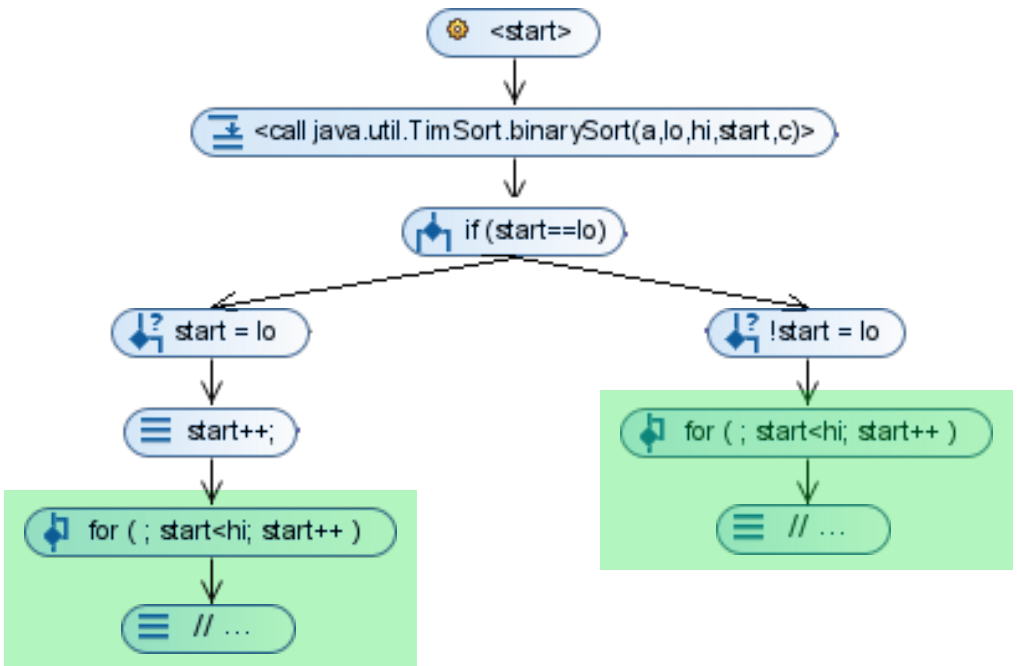
Only choose states with the *same program counter*



(U_1, C_1, φ) (U_2, C_2, φ)

(U^*, C^*, φ)

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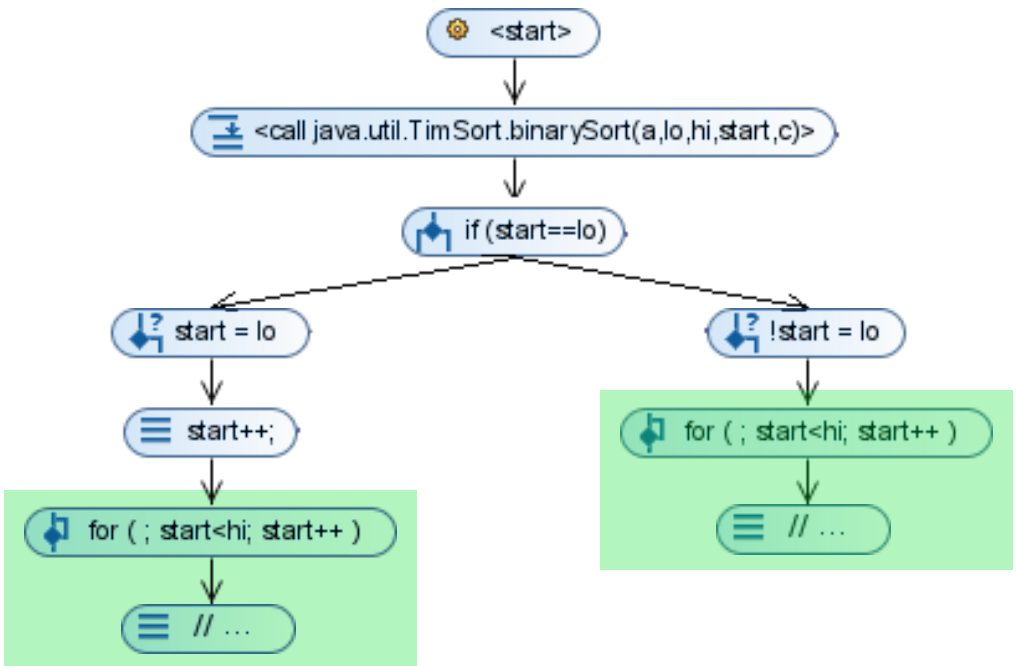

(U_1, C_1, φ) (U_2, C_2, φ)

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Merging Branches in SE Trees: The formal framework

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An SE state represents many *concrete states...*

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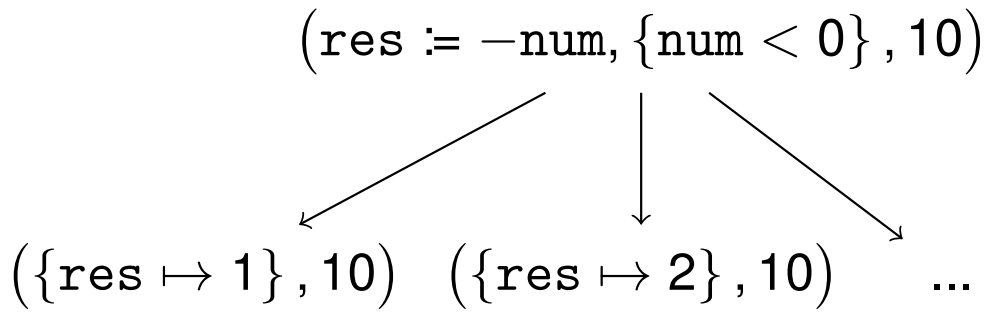
```
1   public int abs(int num) {
2       int res;
3
4       if (num < 0) {
5           res = -num;
6       } else {
7           res = num;
8       }
9
10      return res;
11  }
```



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...and some are “weaker” than others

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Definition

The *concretization function concr* maps an SE state $s = (U, C, \varphi)$ to a set of concrete states, where

$$concr(s) := \left\{ (\sigma', \varphi) : \underbrace{\sigma' = val(K, \sigma; U) \wedge (K, \sigma) \models C}_{\text{All values satisfying the path condition}} \right\}$$

Definition

Let s_1, s_2 be two SE states. We say that s_2 is *weaker* than s_1 (“$s_1 \lesssim s_2$”) if and only if $concr(s_1) \subseteq concr(s_2)$.

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Some simple examples for weakening

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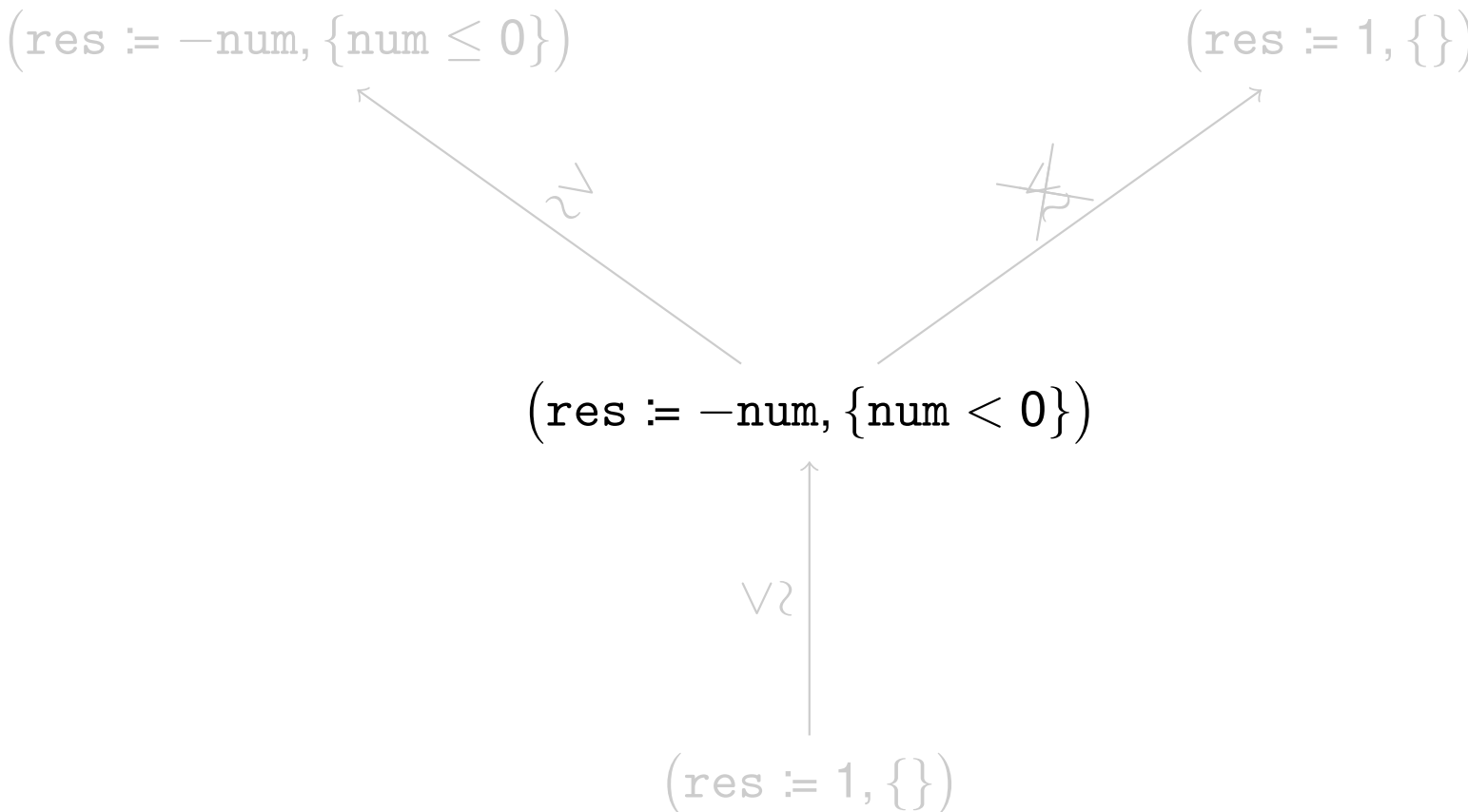

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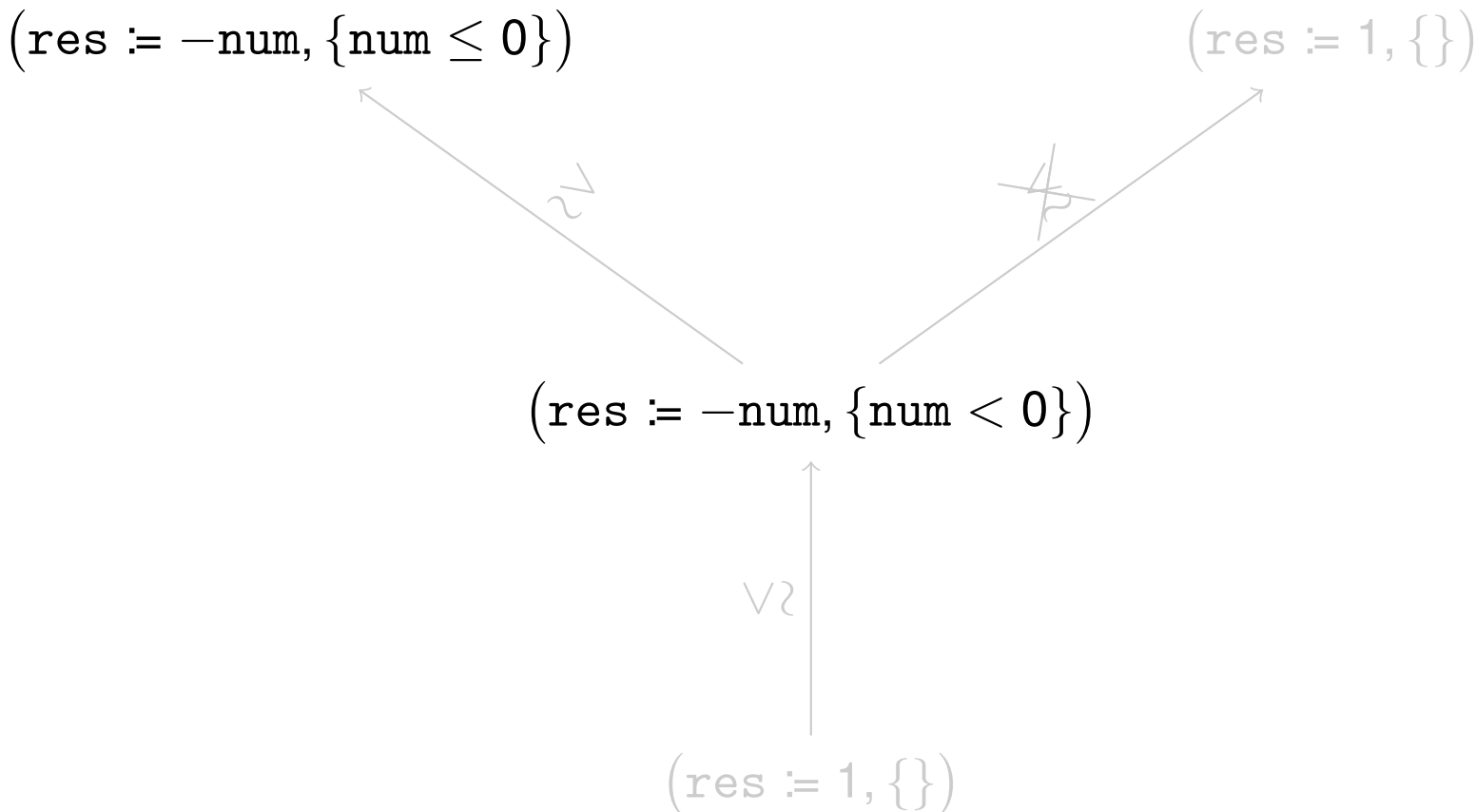

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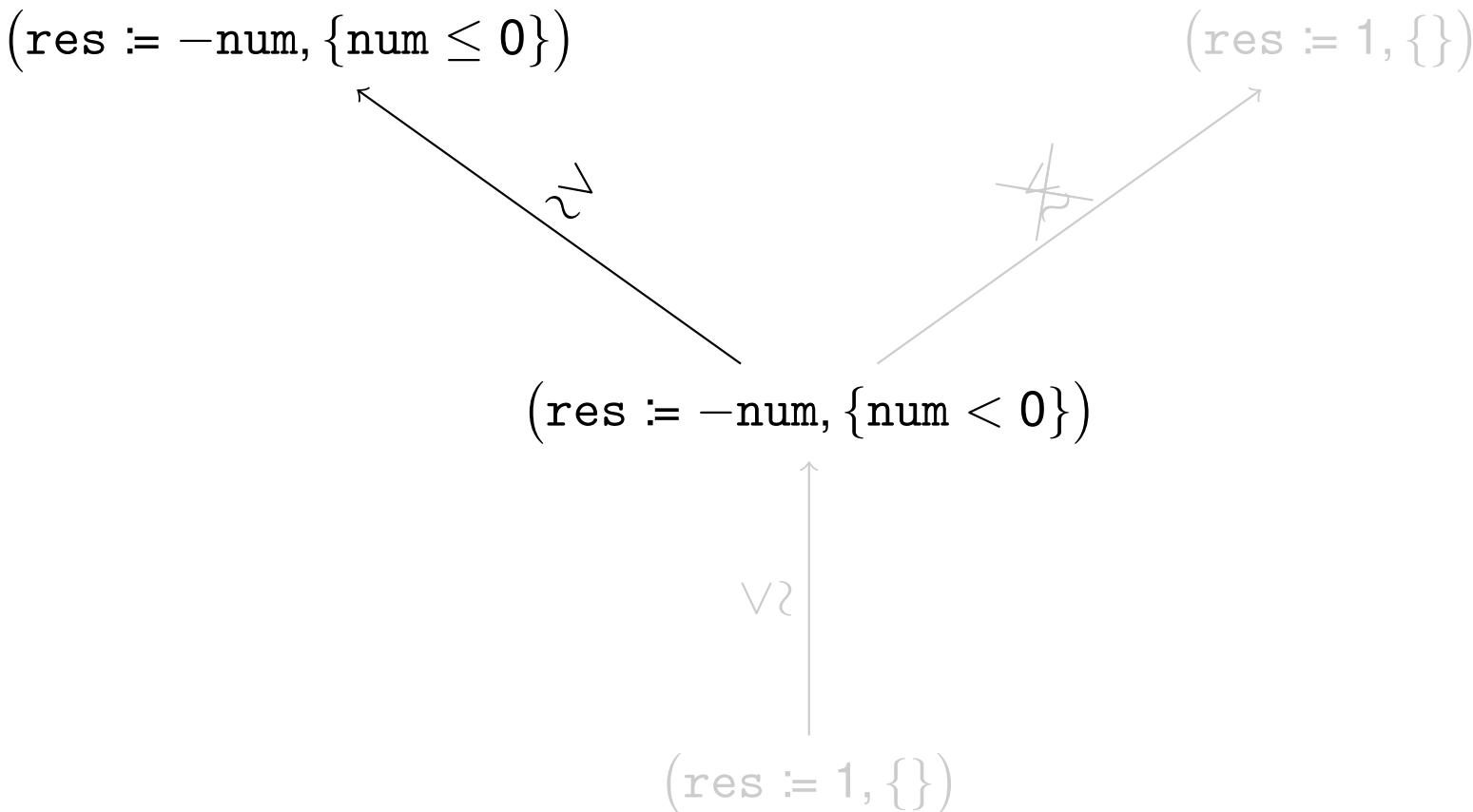

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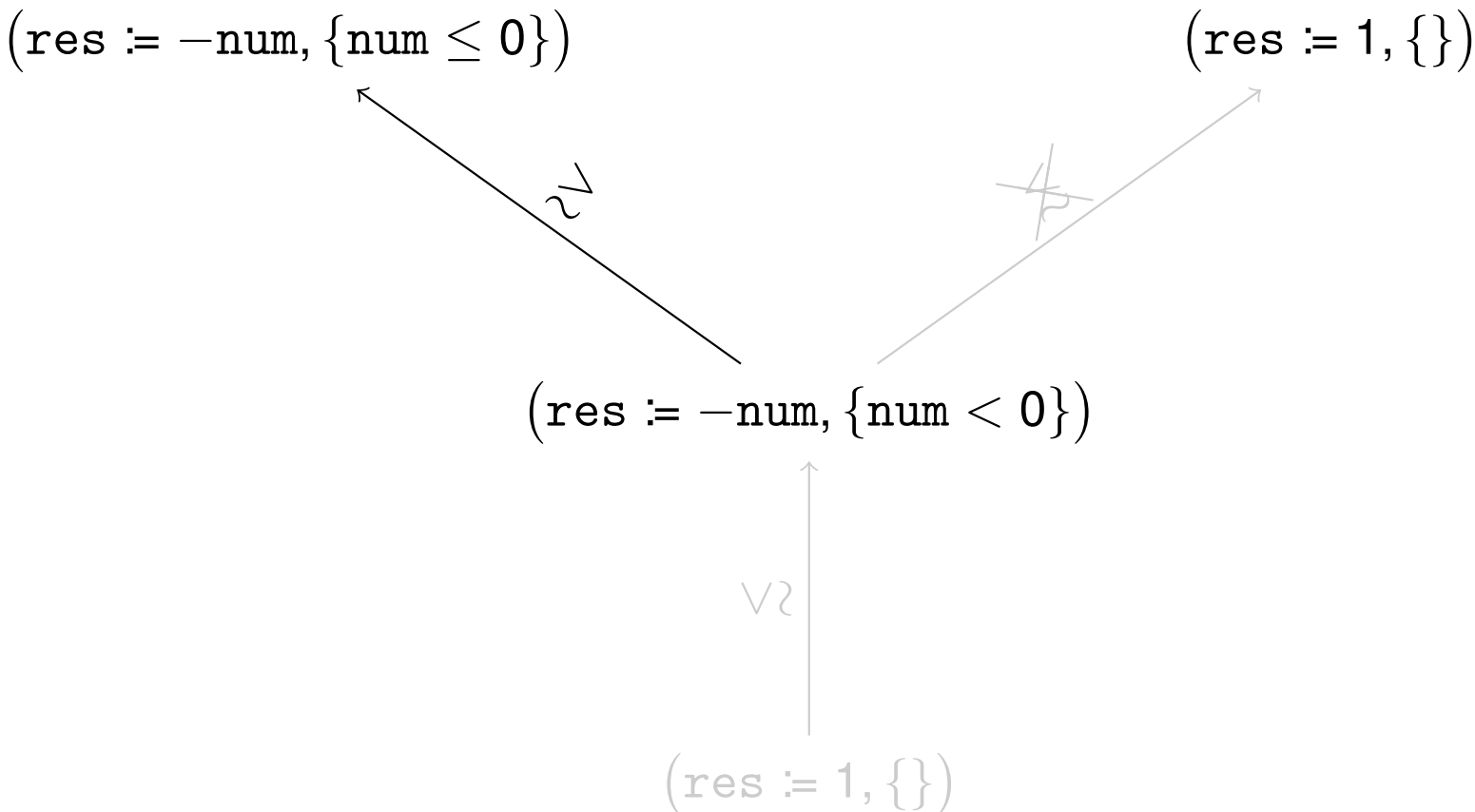

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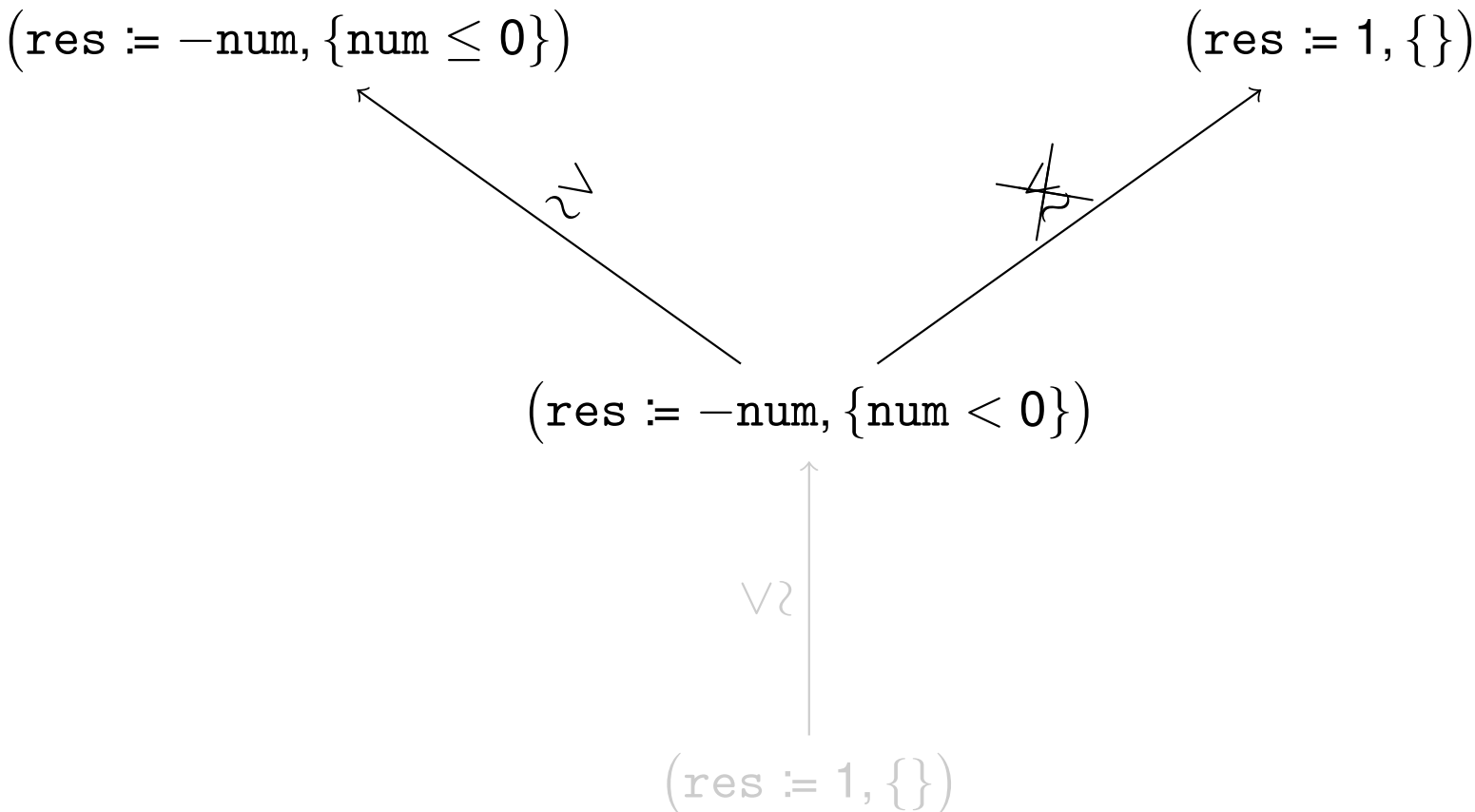

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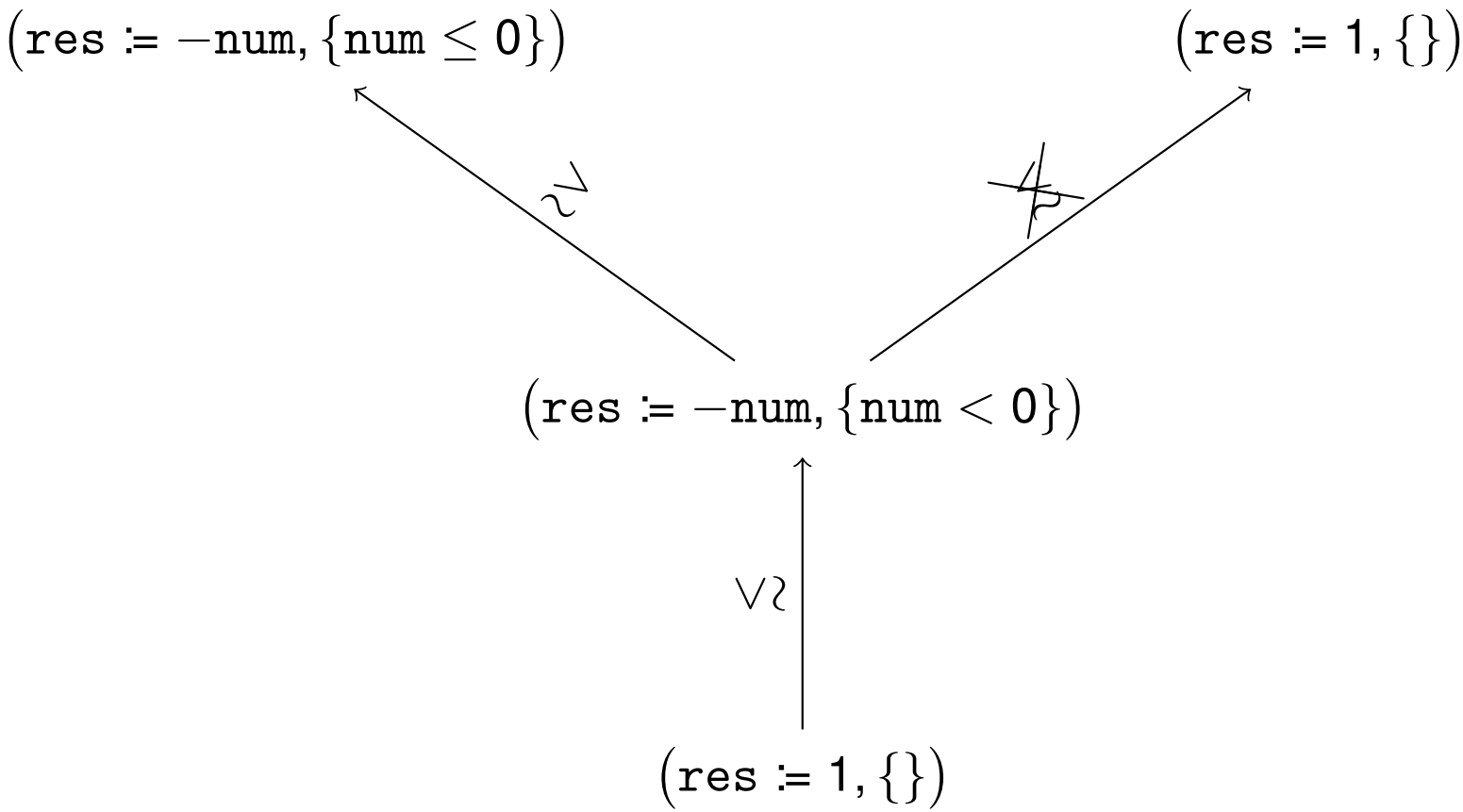

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Closing the gap to abstract interpretation: The general lattice model for SE

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Definition

Let $\dot{\sqcup}$ be an operation on SE states. We call the family of structures

$$\{\mathbb{L}_\varphi\}_{\varphi \in \mathrm{Fml}} := \left\{\left(S_\varphi, \dot{\sqcup}\right)\right\}_\varphi \text{ where}$$

$$S_\varphi := \{(U, C, \varphi) \mid (U, C, \varphi) \text{ is an SE state}\}$$

the *induced family of join-semilattices for* $\dot{\sqcup}$ iff $\dot{\sqcup}$ is *idempotent*, *commutative* and *associative* for states with the *same program counter*, its induced partial order relation $\preceq$ *is consistent with* "$\lesssim$", and both operands $\subseteq$ of the result *have the same program counter*.

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The general lattice model cont'd: Constraints on merge path conditions

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$(U_1, C_1, \varphi) \preceq (U_2, C_2, \varphi)$ implies

- C_2 is logically equivalent to a formula $C \wedge Ax_{\overline{v}} \left[\overline{c} \,/\, \overline{v}\right]$ ($\overline{c}$ are "new" Skolem constants in (U_2, C_2, φ)), where
- $\bigwedge C_1 \rightarrow C$ is provable and
- $\exists \overline{v}; Ax_{\overline{v}}$ is provable

Lemma

If a formula φ does not contain any of the $\overline{c}$ and is true in all models (in a signature with $\overline{c}$) that satisfy C and $Ax_{\overline{v}} \left[\overline{c} \,/\, \overline{v}\right]$, then φ is also true in all models that only satisfy C (in the signature without $\overline{c}$).

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An example for constraints on merge path conditions

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$$(\mathtt{x} \coloneqq 42, C_1, \varphi) \dot{\sqcup} (\mathtt{x} \coloneqq 17, C_2, \varphi) =$$

$$\left(\mathtt{x} \coloneqq c, \left\{ \left(\bigwedge C_1 \vee \bigwedge C_2 \right), c \geq 0 \right\}, \varphi \right)$$

$$(\mathtt{x} \coloneqq 42, C_1, \varphi) \preceq \left(\mathtt{x} \coloneqq c, \underbrace{\left\{ \left(\bigwedge C_1 \vee \bigwedge C_2 \right), c \geq 0 \right\}}_{\equiv C \wedge Ax_{\overline{v}}[\overline{c}/\overline{v}]}, \varphi \right)$$

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Merging with join-semilattices is sound

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Theorem

Let $\varphi \in \mathsf{Fml}$ *be a formula and* $\dot{\sqcup}$ *be a join operation of an induced join-semilattice* $\mathbb{L}_\varphi$ *. Then, merging two SE states*

$$s_i = (U_i, C_i, \varphi)\,, \quad i = 1, 2,$$

to a state

$$s^* = s_1 \dot{\sqcup} s_2$$

is sound*, i.e. if* s^* *is valid, then both* s_1 *and* s_2 *are valid.*

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$2\frac{1}{2}$ popular merge techniques

Maximum precision with *if-then-else*

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$$(\ldots \,||\mathtt{x} := 42||\, \ldots, C_1, \varphi) \qquad (\ldots \,||\mathtt{x} := -17||\, \ldots, C_2, \varphi)$$

$$(\ldots \,||\mathtt{x} := \mathit{if}\ \left(\bigwedge C_1\right)\ \mathit{then}\ (42)\ \mathit{else}\ (-17)\,||\, \ldots, \left\{\bigwedge C_1 \vee \bigwedge C_2\right\}, \varphi)$$

Maximum precision with *if-then-else*

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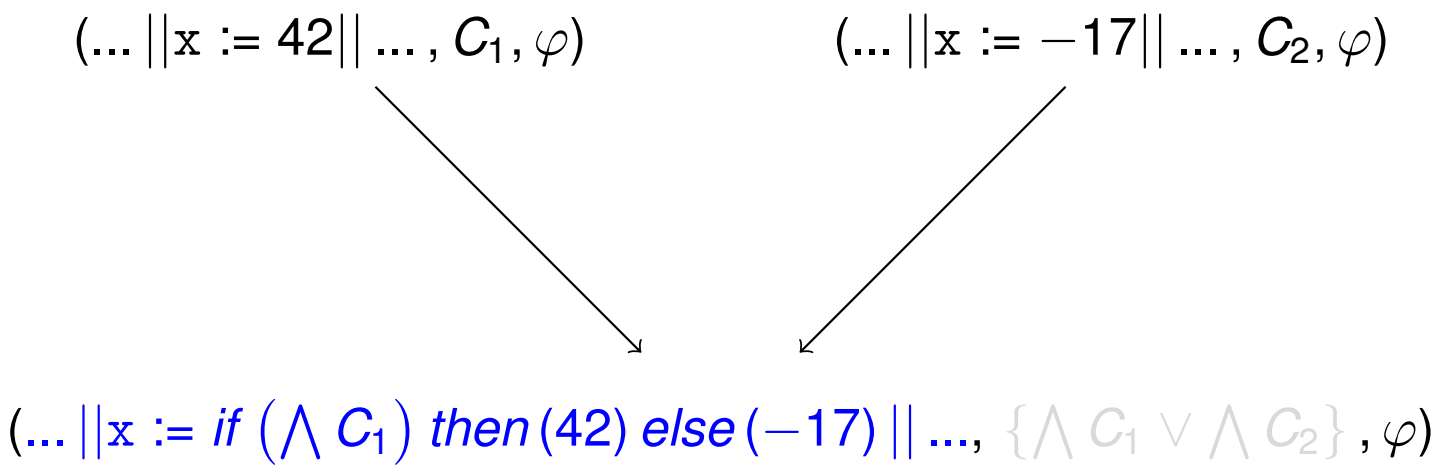

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Maximum precision with *if-then-else*

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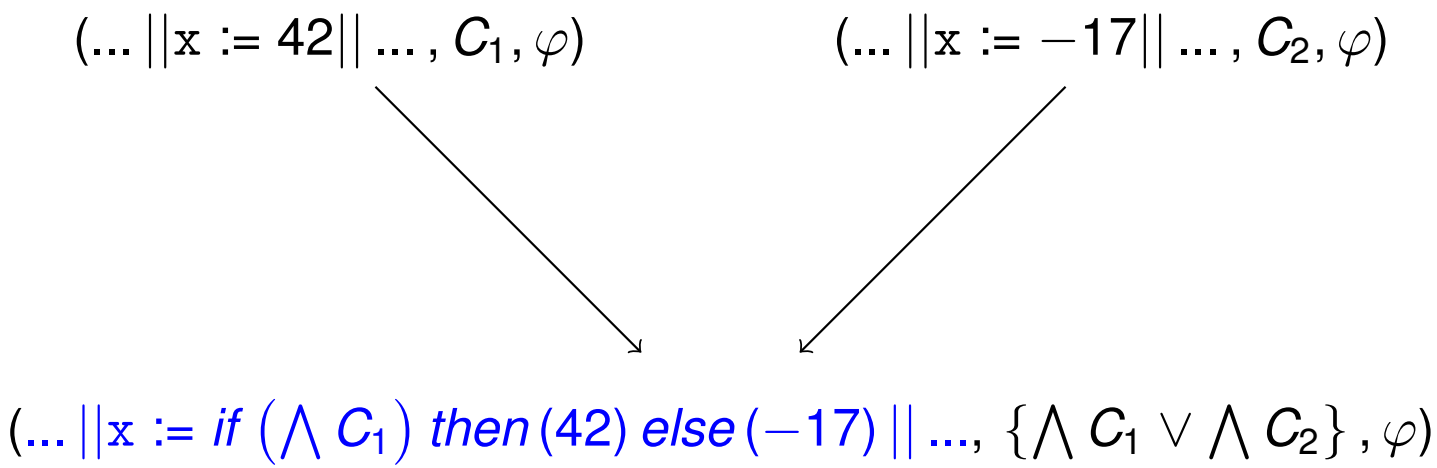

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Symbolic Execution meets Abstract Interpretation

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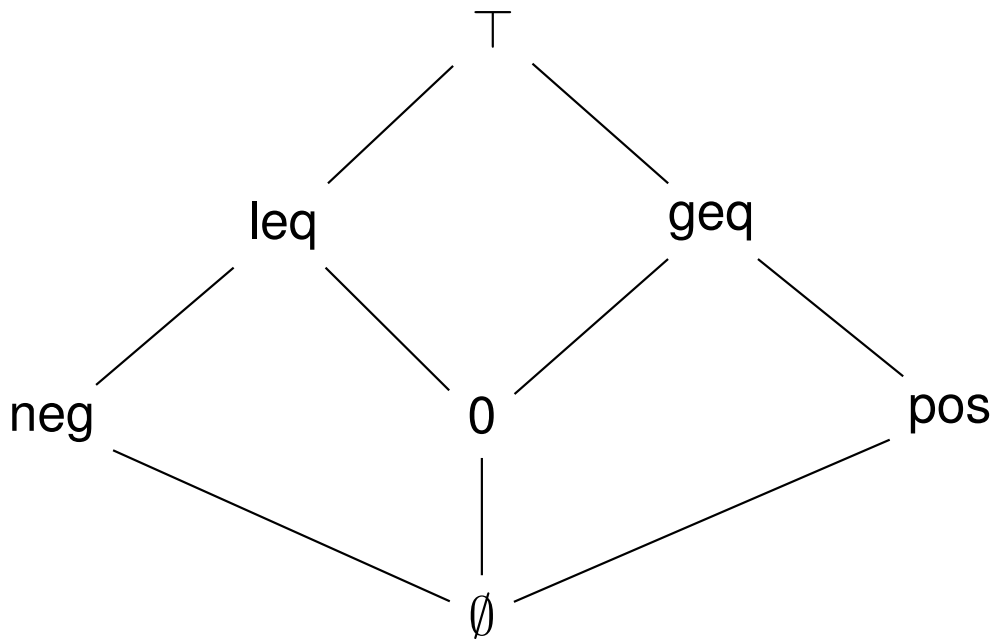

$$\gamma(\top) = \mathbb{Z}$$
$$\gamma(\text{leq}) = \{i \in \mathbb{Z} | i \leq 0\}$$
$$\gamma(\text{geq}) = \{i \in \mathbb{Z} | i \geq 0\}$$
$$\gamma(\text{neg}) = \{i \in \mathbb{Z} | i < 0\}$$
$$\gamma(\text{pos}) = \{i \in \mathbb{Z} | i > 0\}$$
$$\gamma(0) = \{0\}$$
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Example from [BHW09]

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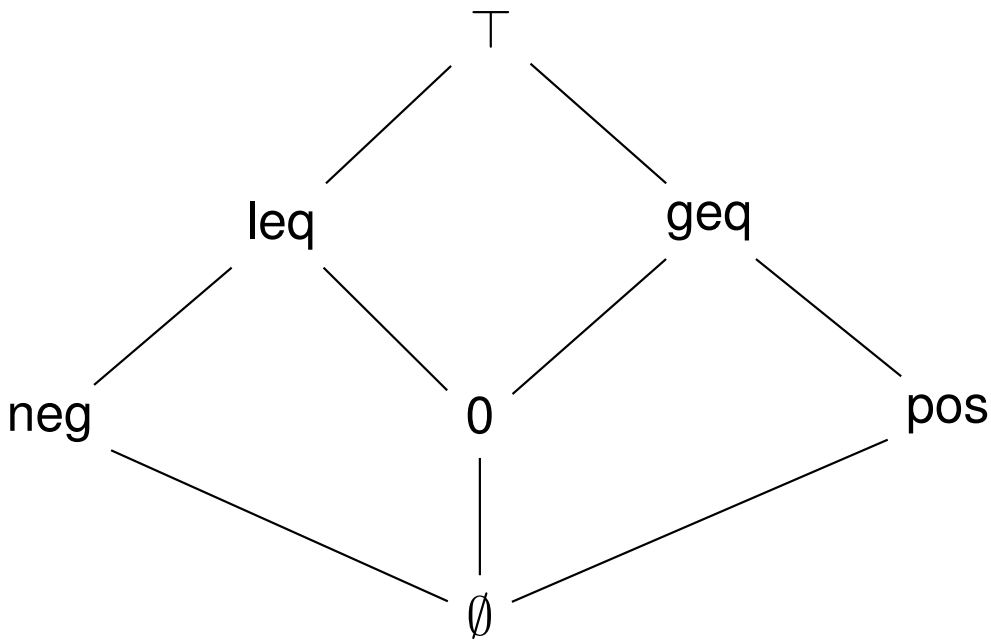

$$
\begin{aligned}
\gamma(\top) &= \mathbb{Z} \\
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\end{aligned}
$$

Example from [BHW09]

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$$(\ldots \,||\mathtt{x} := 0||\, \ldots, C_1, \varphi) \qquad (\ldots \,||\mathtt{x} := -17||\, \ldots, C_2, \varphi)$$

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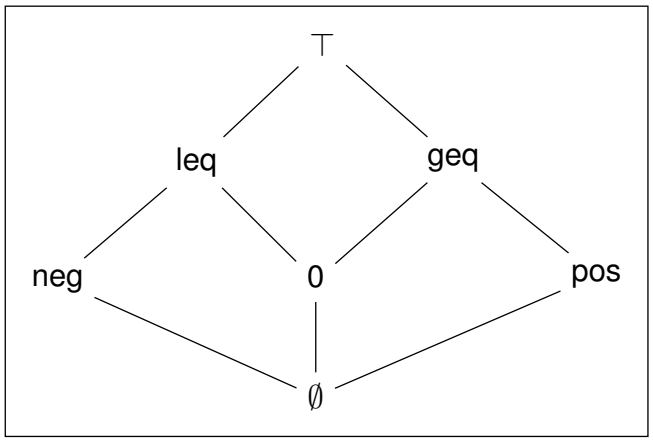

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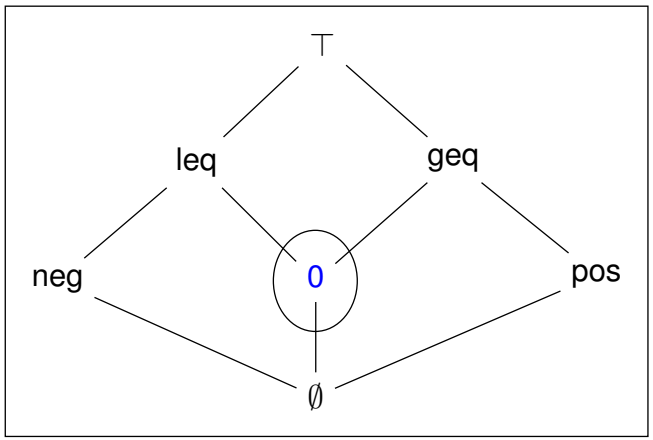

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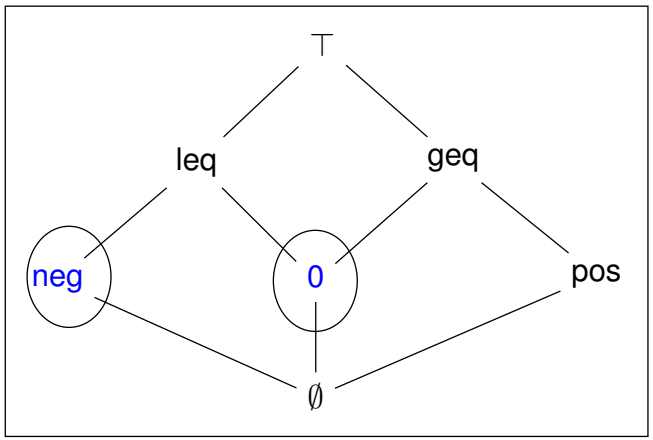

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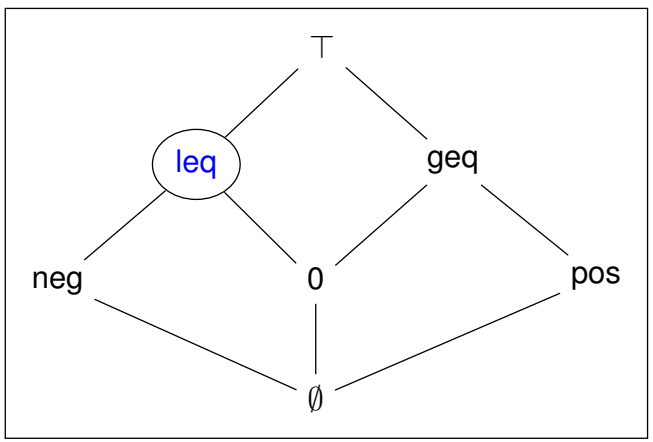

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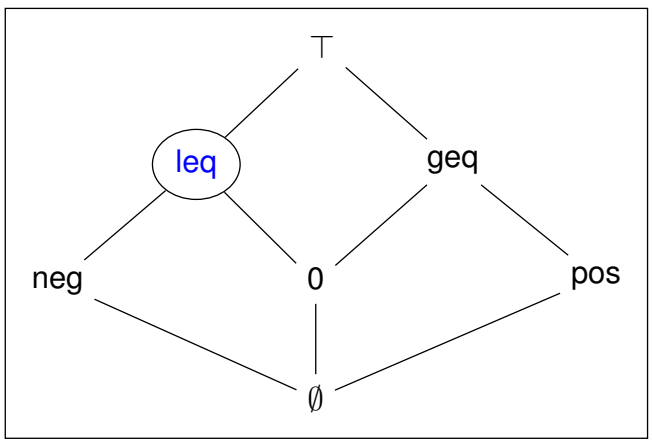

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Predicate Abstraction facilitates the ad-hoc construction of abstract domains tailored to a *specific problem*

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$preds := \{\square \geq x,\ \square \geq 0\}$

```
1   // prove: result >= 0
2   if (y < x) {
3       int tmp = x;
4       x = y;
5       y = tmp;
6   }
7
8   return y - x;
```


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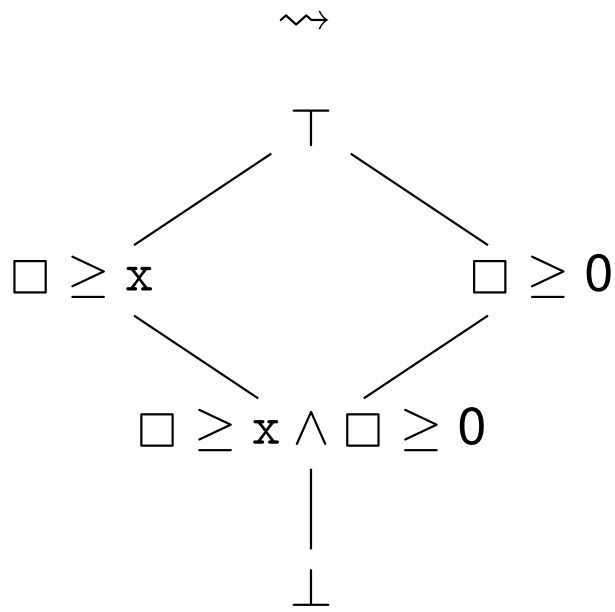

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State merging in the SE workflow

A semi-automatic approach: *Merge annotations* in block contracts

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```
/*@ join_proc "JoinByIfThenElse"; @*/
{
    if (current.parent == null}
        root = newParent;
        newParent.parent = null;
    } else {
        current.parent.right = newParent;
        newParent.parent = current.parent;
    }
}
current.parent = newParent;
//...
```

A semi-automatic approach: *Merge annotations* in block contracts

TECHNISCHE UNIVERSITÄT DARMSTADT

```
/*@ join_proc "JoinByIfThenElse"; @*/
{
                                                                    ;
}
c
// ...
```

Supported join_proc values

- "JoinByFullAnonymization"
- "JoinByIfThenElse"
- "JoinByIfThenElseAntecedent"
- "JoinBySignLatticeAbstraction"

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Evaluation, Conclusion and Outlook

Results of the TimSort case study: Merging can shorten proofs significantly...

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Value measured	With merging	Without merging
Nodes	88,333	303,716
Branches	324	632
Automode time	33min	28min
Merge Rule apps	6	0

⇒ **71%** improvement in #nodes

Timsort.gallopLeft

Value measured	With merging	Without merging
Nodes	63,309	279,156
Branches	1,576	1,804
Automode time	32min	41min
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⇒ **77%** improvement in #nodes

Timsort.mergeAt

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Results of the TimSort case study: ...facilitates symbolic execution of previously infeasible methods...

TECHNISCHE UNIVERSITÄT DARMSTADT

Value measured	With merging	Without merging
Nodes	460,410	N/A
Branches	2,310	N/A
Automode time	2h 29min	N/A
Merge Rule apps	5	0

Timsort.mergeHi

Value measured	With merging	Without merging
Nodes	1,455,919	N/A
Branches	2,906	N/A
Automode time	14h 20min	N/A
Merge Rule apps	12	0

Timsort.mergeLo

Results of the TimSort case study: ...and makes some complicated block contracts superfluous.

TECHNISCHE UNIVERSITÄT DARMSTADT

```
/*@ normal_behavior
  @ ensures
  @       (i == (\bigint)stackSize-3 ==>
  @          (    runLen[(\bigint)i+1]  == \old(runLen[(\bigint)i+2])
  @           && runBase[(\bigint)i+1] == \old(runBase[(\bigint)i+2])))
  @   &&
  @       (i == (\bigint)stackSize-2 ==>
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  @           && runBase[(\bigint)i+1] == \old(runBase[(\bigint)i+1])));
  @ assignable
  @   runBase[i+1], runLen[i+1];
  @*/
```



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//@ join_proc "JoinByIfThenElse";
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Examples of Ongoing and Future Work

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```
/*@ merge contract;
  @ \method "Predicate Abstraction";
  @ \predicates (v, v >= 0),
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if (y < x) {
    int tmp = x;
    x = y;
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return y - x;
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Fully fledged block contracts for state merging

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/*@ merge contract;
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Automatic inference of abstraction predicates

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Automatic inference of abstraction predicates

A general framework for merging SE states with KeY

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- ✓ Parametric merge rule with soundness result
- ✓ General lattice framework for admissible join operations
- ✓ Instantiations for the two most prominent join operations
- ✓ Implementation for KeY theorem prover
- ✓ Evaluation with Timsort: Merging can significantly reduce size of proof trees

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